Calhoun: The NPS Institutional Archive
DSpace Repository

Theses and Dissertations 1. Thesis and Dissertation Collection, all items

1969-06

Computer simulation of finite amplitude standing waves in rigid-walled ducts.

Kadlick, Richard Mark

Monterey, California. Naval Postgraduate School

http://hdl.handle.net/10945/24639

This publication is a work of the U.S. Government as defined in Title 17, United States Code, Section 101. Copyright protection is not available for this work in the United States.

Downloaded from NPS Archive: Calhoun

Calhoun is the Naval Postgraduate School's public access digital repository for research materials and institutional publications created by the NPS community. Calhoun is named for Professor of Mathematics Guy K. Calhoun, NPS's first appointed -- and published -- scholarly author.

Dudley Knox Library / Naval Postgraduate School
411 Dyer Road / 1 University Circle
Monterey, California USA 93943

http://www.nps.edu/library

NPS ARCHIVE
1969
KADLICK, R.

COMPUTER SIMULATION OF FINITE AMPLITUDE STANDING WAVES IN RIGID-WALLED DUCTS

by

Richard Mark Kadlick

DUDLEY KNOX LIBRARY
NAVAL POSTGRADUATE SCHOOL
MONTEREY, CA 93943-5101

United States
Naval Postgraduate School

THESIS

COMPUTER SIMULATION OF FINITE-AMPLITUDE STANDING WAVES IN RIGID-WALLED DUCTS

by

Richard Mark Kadlick

T132203

June 1969

This document has been approved for public release and sale; its distribution is unlimited.

DUDLEY KNOX LIBRARY
NAVAL POSTGRADUATE SCHOOL
MONTEREY, CA 93943-5101

Computer Simulation of Finite-Amplitude Standing Waves in Rigid-Walled Ducts

by

Richard Mark Kadlick
Lieutenant (junior grade), United States Navy
B.S., United States Naval Academy, 1968

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN PHYSICS

from the

NAVAL POSTGRADUATE SCHOOL
June 1969

NPS ARCHIVE ~~[illegible] K107~~ c.1
1969
ADLICK, R.

ABSTRACT

The Coppens-Sanders theory for the one-dimensional, nonlinear, acoustic wave equation with dissipative term describing the viscous and thermal energy losses encountered in a rigid-walled, closed tube of large length-to-diameter ratio was applied to finite-amplitude standing waves by the use of the Fast Fourier Transform. Computer programs were written to determine the amplitudes and phases of the first 255 harmonics. Curves of harmonic distortion as a function of the strength parameter were found to be in excellent agreement with available experimental data, to agree with the Coppens-Sanders perturbation analysis, and to extend the theoretically describable régime closer to strengths leading to the formation of the shock front.

Library
U.S. Naval Postgraduate School
Monterey, California 93940

TABLE OF CONTENTS

I. INTRODUCTION -- 13

II. THEORY -- 14

III. RESULTS, DISCUSSION, AND CONCLUSIONS -- 20

APPENDIX A The Fast Fourier Transform -- 27

APPENDIX B Perturbation Solution of Eq. 1.1 -- 31

COMPUTER PROGRAM -- 33

BIBLIOGRAPHY -- 36

INITIAL DISTRIBUTION LIST -- 37

FORM DD 1473 -- 39

LIST OF TABLES

I. Computer Program Symbology ---------------------------------- 33

LIST OF FIGURES

1. Flow Diagram for Solving Eq. 1.1 ------------------------------ 19

2. Strength Curves -- 22

3. Response Curves, $Mb/\delta_1 = 0.2$ ---------------------------------- 23

4. Response Curves, $Mb/\delta_1 = 0.4$ ---------------------------------- 24

5. Response Curves, $Mb/\delta_1 = 0.6$ ---------------------------------- 25

6. Response Curves, $Mb/\delta_1 = 0.8$ ---------------------------------- 26

7. Waveform for Analysis -- 28

8. Reconstructed Waveform Using Fourier Transform --------------- 29

9. Reconstructed Waveform Using Fast Fourier Transform ---------- 30

LIST OF SYMBOLS

ρ,ρ_o = instantaneous density, ambient density

ω = driving (angular) frequency

ω_r = resonance frequency for the fundamental

$\Delta\omega$ = $\omega - \omega_r$

c_n = phase speed in the tube associated with the n^{th} harmonic

c_o = $(\partial p/\partial \rho)^{1/2}_{\rho=\rho_o}$ for dissipationless propagation (linearized theory)

k = ω/c

L = length of the tube

x = Lagrangian coordinate measured in the x direction from the driven end

ξ = particle displacement in the x direction

u = particle speed in the x direction, $(\partial\xi/\partial t)$

p = acoustic pressure

p_n = acoustic pressure for the n^{th} harmonic

P_n = amplitude of p_n

A_n = $P_n/M\rho_o c_o^2$

ϕ_n = phase angle associated with the temporal part of the acoustic pressure

δ = dissipation parameter

δ_n = dissipation parameter for the n^{th} frequency component

$\Box_L^2$ = D'Alambertian in Lagrangian coordinates (one spatial dimension)

$\mathcal{D}_{Ln}$ = dissipation operator for the n^{th} frequency component

$(\delta_n/\omega_n)\partial^3/\partial_x^2\partial t - \delta_n\partial^2/\partial x^2$

M = a Mach number (See Eq. 2.3)

B/A = parameter of nonlinearity $\rho_o(\partial^2 p/\partial\rho^2)_{\rho=\rho_o}$

b = 1 + 1/2 (B/A)

H_n = amplitude operator (See Eq. 2.13)

θ_n = phase operator (See Eq. 2.14)

B_n = amplitude associated with p_n^2 (See Eq. 2.9)

Γ_n = phase angle associated with B_n

ε = $(\rho_o c_o^2)^{-1} (\eta_B + 4\eta/3)$

η_B = coefficient of bulk viscosity

η = coefficient of shear viscosity

ACKNOWLEDGEMENT

The author would like to express his appreciation to Professor Alan B. Coppens for his generous aid and encouragement.

Thanks are due to Professor H. A. Titus for his help on the use of the Fast Fourier Transform.

The IBM-360 Computer was used for this research and the help of the staff of the facility is gratefully acknowledged.

This work was supported in part by the Office of Naval Research.

I. INTRODUCTION

Theoretical investigations of finite-amplitude waves have been characterized by various approximation and perturbation techniques, but more significantly by the postulation of various models of the absorptive mechanisms [1,2,3,4]. Coppens and Sanders [5] have investigated finite-amplitude standing waves for rigid-walled tubes of large length-to-diameter ratio by a perturbation method and have proposed that the dominant energy losses are produced by the viscous effects of the fluid at the walls and the thermal losses due to the wall material. They have shown that an approximate wave equation for the process is

$$\sum_{n=1}^{\infty}\left(\square_L^2 + \mathcal{D}_{Ln}\right)u_n \doteq b\frac{\partial}{\partial x}\frac{\partial}{\partial t}\left(\frac{\partial \xi}{\partial t}\right)^2 \qquad (1.1)$$

where $u = \sum_{n=1}^{\infty} u_n$ and the subscript n represents the n^{th} harmonic of the waveform. The n^{th} frequency component of $\mathcal{D}_L$ is

$$\mathcal{D}_{Ln} = \frac{\delta_n}{\omega_n}\frac{\partial^3}{\partial x^2 \partial t} - \delta_n\frac{\partial^2}{\partial x^2} \qquad (1.2)$$

where δ_n is the dissipation parameter [3,5] for the n^{th} frequency component defined by $\delta_n = \delta_1/\sqrt{n}$. The process has two characteristic parameters, one related to the strength of the standing wave and the other related to its frequency.

The purpose of this thesis is extend the Coppens-Sanders approach by a "Fast Fourier" analysis technique. By an iterative method, the amplitudes and phases of the various harmonics are obtained for different values of the strength and frequency parameters and are compared with available experimental and theoretical results.

II. THEORY

In this section a Fourier-synthesis approach is presented for the one-dimensional, nonlinear, dissipative, acoustic wave equation based on the theory of Coppens and Sanders as applied to finite-amplitude standing waves in a rigid-walled, closed tube of large length-to-diameter ratio.

We begin with Eq. 1.1 which describes the process in question. Notice that

$$p = \sum_{n=1}^{\infty} p_n = -\rho_o c_o^2 \left(\frac{\partial \xi}{\partial x}\right) \qquad (2.1)$$

so that Eq. 1.1 may be written as

$$-\sum_{n=1}^{\infty} \left(\Box_L^2 + \mathcal{D}_{Ln}\right) p_n = \frac{b}{\rho_o c_o^2} \frac{\partial^2}{\partial x^2} \left(p^2\right) . \qquad (2.2)$$

Assume that p may be written as a Fourier series

$$p = \sum_{n=1}^{\infty} P_n \cos nk\left(L - x\right) \sin\left(n\omega t + \phi_n\right) \qquad (2.3)$$

where $P_n = M\rho_o c_o^2 A_n$.

Substitution for $\Box_L^2$ and $\mathcal{D}_{Ln}$ into the left-hand side of Eq. 2.2 results in

$$-\sum_{n=1}^{\infty} \left[\frac{\partial^2}{\partial x^2} - \frac{1}{c_o^2}\frac{\partial^2}{\partial t^2} + \frac{\delta_n}{\omega_n}\frac{\partial^2}{\partial x^2}\frac{\partial}{\partial t} - \delta_n \frac{\partial^2}{\partial x^2}\right] p_n . \qquad (2.4)$$

Upon substitution for δ_n and ω_n, Exp. 2.4 becomes

$$-\sum_{n=1}^{\infty} \left[\left(1 - \frac{\delta_1}{n^{\frac{1}{2}}}\right)\frac{\partial^2}{\partial x^2} - \frac{1}{c_o^2}\frac{\partial^2}{\partial t^2} + \frac{\delta_1}{n^{\frac{3}{2}}\omega}\frac{\partial^2}{\partial x^2}\frac{\partial}{\partial t}\right] p_n . \qquad (2.5)$$

Notice that $\frac{\partial^2\mathcal{P}_n}{\partial x^2}=-(nk)^2\mathcal{P}_n$ and $\frac{\partial^2\mathcal{P}_n}{\partial t^2}=-(n\omega)^2\mathcal{P}_n$

so that Exp. 2.5 may be written as

$$-\sum_{n=1}^{\infty}(nk)^2\left[-\left(1-\frac{\delta_1}{n^{\frac{1}{2}}}\right)+\frac{\omega^2}{k^2c_0^2}-\frac{\delta_1}{n^{\frac{3}{2}}\omega}\frac{\partial}{\partial t}\right]\mathcal{P}_n\ . \tag{2.6}$$

Define $\omega=\omega_r+\Delta\omega$ where $\Delta\omega \ll \omega_r$, and for the fundamental $k\equiv\frac{\omega_r}{c_1}\doteq\frac{\pi}{L}$ where the approximation is valid for weak absorption. The approximation may be made that

$$\frac{\omega^2}{k^2c_0^2}=\frac{c_1^2}{c_0^2}+\frac{c_1^2}{c_0^2}\left(\frac{2\Delta\omega}{\omega_r}\right)\ . \tag{2.7}$$

Define $(c_1/c_0)^2=1-\delta_1$ and neglect second-order terms; Exp. 2.6 may then be written as

$$-\delta_1\sum_{n=1}^{\infty}(nk)^2\left[\frac{1}{n^{\frac{1}{2}}}-1+\frac{2\Delta\omega}{\omega_r\delta_1}-\frac{1}{n^{\frac{3}{2}}\omega}\right]\mathcal{P}_n\ . \tag{2.8}$$

The right-hand side of Eq. 2.2 contains the quantity p^2. This involves multiplication of two infinite series. Terms of the form $\cos p(L-x)\ \sin(q\omega t+\phi_q)$ for $p\neq q$ may be neglected [5], so that p^2 may be approximated by

$$\mathcal{P}^2=\frac{(M\rho_0c_0^2)^2}{2}\sum_{n=1}^{\infty}B_n\cos nk(L-x)\sin(n\omega t+\Gamma_n)\ . \tag{2.9}$$

Then we have $\frac{\partial^2(\mathcal{P}^2)}{\partial x^2}=-(nk)^2\mathcal{P}^2$ and Eq. 2.2 may be written as

$$\delta_1\sum_{n=1}^{\infty}\left[\frac{1}{n^{\frac{1}{2}}}-1+\frac{2\Delta\omega}{\delta_1\omega_r}-\frac{1}{n^{\frac{3}{2}}\omega}\frac{\partial}{\partial t}\right]A_n\cos nk(L-x)\sin(n\omega t+\phi_n) \tag{2.10}$$

$$=\frac{Mb}{2}\sum_{n=1}^{\infty}B_n\cos nk(L-x)\sin(n\omega t+\Gamma_n)\ .$$

If attention is limited to the rigid end at x = L, Eq. 2.10 may be considerably simplified to

$$\left\{\sum_{n=1}^{\infty}\left[\frac{1}{n^{1/2}}-1+\frac{2\Delta\omega}{\delta_1\omega_r}\right]\sin(n\omega t+\phi_n)-\frac{1}{n^{1/2}}\cos(n\omega t+\phi_n)\right\}A_n = \frac{Mb}{2\delta_1}\sum_{n=1}^{\infty}B_n\sin(n\omega t+\Gamma_n). \quad (2.11)$$

This becomes

$$\sum_{n=1}^{\infty}A_n\sin(n\omega t+\phi_n+\theta_n) = \frac{Mb}{2\delta_1}\sum_{n=1}^{\infty}H_nB_n\sin(n\omega t+\Gamma_n) \quad (2.12)$$

where

$$H_n=\left[\left(\frac{1}{n^{1/2}}-1+\frac{2\Delta\omega}{\omega_r\delta_1}\right)^2+\left(\frac{1}{n^{1/2}}\right)^2\right]^{-\frac{1}{2}} \quad (2.13)$$

and

$$\theta_n=\tan^{-1}\frac{-\frac{1}{n^{1/2}}}{-1+\frac{1}{n^{1/2}}+\frac{2\Delta\omega}{\omega_r\delta_1}}\,. \quad (2.14)$$

The parameters relating to the strength and frequency response of the standing wave are Mb/δ_1 and $\frac{2\Delta\omega}{\omega_r\delta_1}$, respectively.

An iterative solution to Eq. 1.1 may be obtained by successive application of Eqs. 2.12, 2.13, and 2.14 to an initial set of A_n's and ϕ_n's. The necessary procedure is shown as a flow diagram in Fig. 1. A discussion of the Fast Fourier Transform and the reasons for its use herein are found in Appendix A. Notice that any D.C. terms which appear in the procedure have to be set equal to zero since the theory excludes these terms.

A source of computational difficulty arises when the frequency parameter $\frac{2\Delta\omega}{\omega_r\delta_1}$ becomes unity. For this case Eq. 2.13 reduces to $H_n = \sqrt{n/2}$, a result which causes the computer solution to become unstable because of the strong excitation of the higher harmonics. The reason for this difficulty may be seen as follows: the resonance frequencies for the classical nondissipative case are defined by $\omega_n = n\omega_c$, where $\omega_c \doteq c_o \frac{\pi}{L}$. For linearized propagation in a duct, resonance frequencies for exciting each harmonic would be given by $\omega_n' = n\omega' = n\frac{\pi}{L} c_n$, where $c_n = c_o\ (1 - \delta_n/2)$. As $n \to \infty$, we have the limits $c_n \to c_o$ and $\omega' \doteq \omega_c$. For the case of large n then, since $\omega = \omega_r + \Delta\omega$, the value of the response parameter corresponding to the predicted resonances in the linearized theory is given by $\frac{2\Delta\omega}{\omega_r\delta_1} = 1$; thus, the higher harmonics are optimally excited and problems will arise in the computer program, which admits only the first 255 harmonics.

Notice that the bulk losses have been neglected up to this point. If these terms are retained $\mathcal{D}_{Ln}$ may be rewritten [5] as

$$\mathcal{D}_{Ln} \doteq \left[\frac{\delta_n}{\omega_n} + \epsilon \right] \frac{\partial^3}{\partial t \partial x^2} - \delta_n \frac{\partial^2}{\partial x^2} \quad . \tag{2.15}$$

In most experimental configurations the value of ε is usually so small that it may be neglected when compared to δ_n/ω_n. Injection of an unrealistically large value of ε does alleviate, to some extent, the instabilities in the neighborhood of $\frac{2\Delta\omega}{\omega_r\delta_1} = 1$. Equations 2.13 and 2.14 become

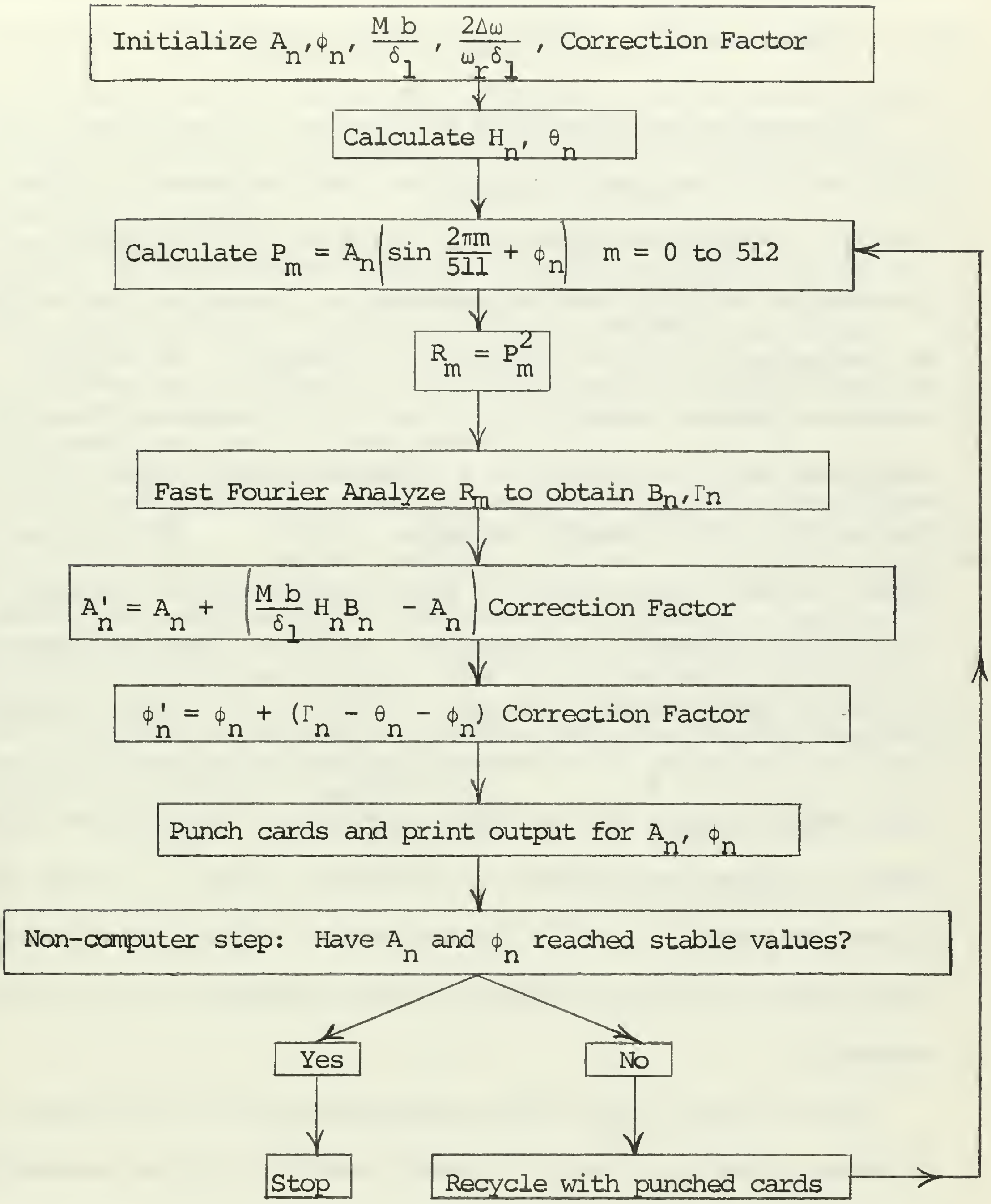
$$H_n = \left[\left(\frac{1}{n^{1/2}} - 1 + \frac{2\Delta\omega}{\omega_r\delta_1} \right)^2 + \left(\frac{1}{n^{1/2}} + \epsilon n \right) \right]^{-\frac{1}{2}} \tag{2.16}$$

and

$$\theta_n = \tan^{-1} \frac{-1/n^{1/2} - \epsilon n}{-1 + \frac{1}{n^{1/2}} + \frac{2\Delta\omega}{\omega_r \delta_1}} \tag{2.17}$$

where $\epsilon = (\rho_o c_o^2)^{-1} (\eta_B + \frac{4}{3}\eta)$.

and H_n would have the asymptotic value of $1/\epsilon$ n as n → ∞. The effectiveness of this modification will be discussed in more detail in Section III.


Initialize $A_n, \phi_n, \frac{M\,b}{\delta_1}, \frac{2\Delta\omega}{\omega_r\delta_1}$, Correction Factor
Calculate $H_n,\ \theta_n$
Calculate $P_m = A_n\left(\sin\frac{2\pi m}{511} + \phi_n\right)$ $m = 0$ to 512
$R_m = P_m^2$
Fast Fourier Analyze R_m to obtain B_n, Γ_n
$A'_n = A_n + \left(\frac{M\,b}{\delta_1} H_n B_n - A_n\right)$ Correction Factor
$\phi'_n = \phi_n + (\Gamma_n - \theta_n - \phi_n)$ Correction Factor
Punch cards and print output for $A_n,\ \phi_n$
Non-computer step: Have A_n and ϕ_n reached stable values?
Yes
No
Stop
Recycle with punched cards

Flow Diagram for Solving Eq. 1.1

Figure 1

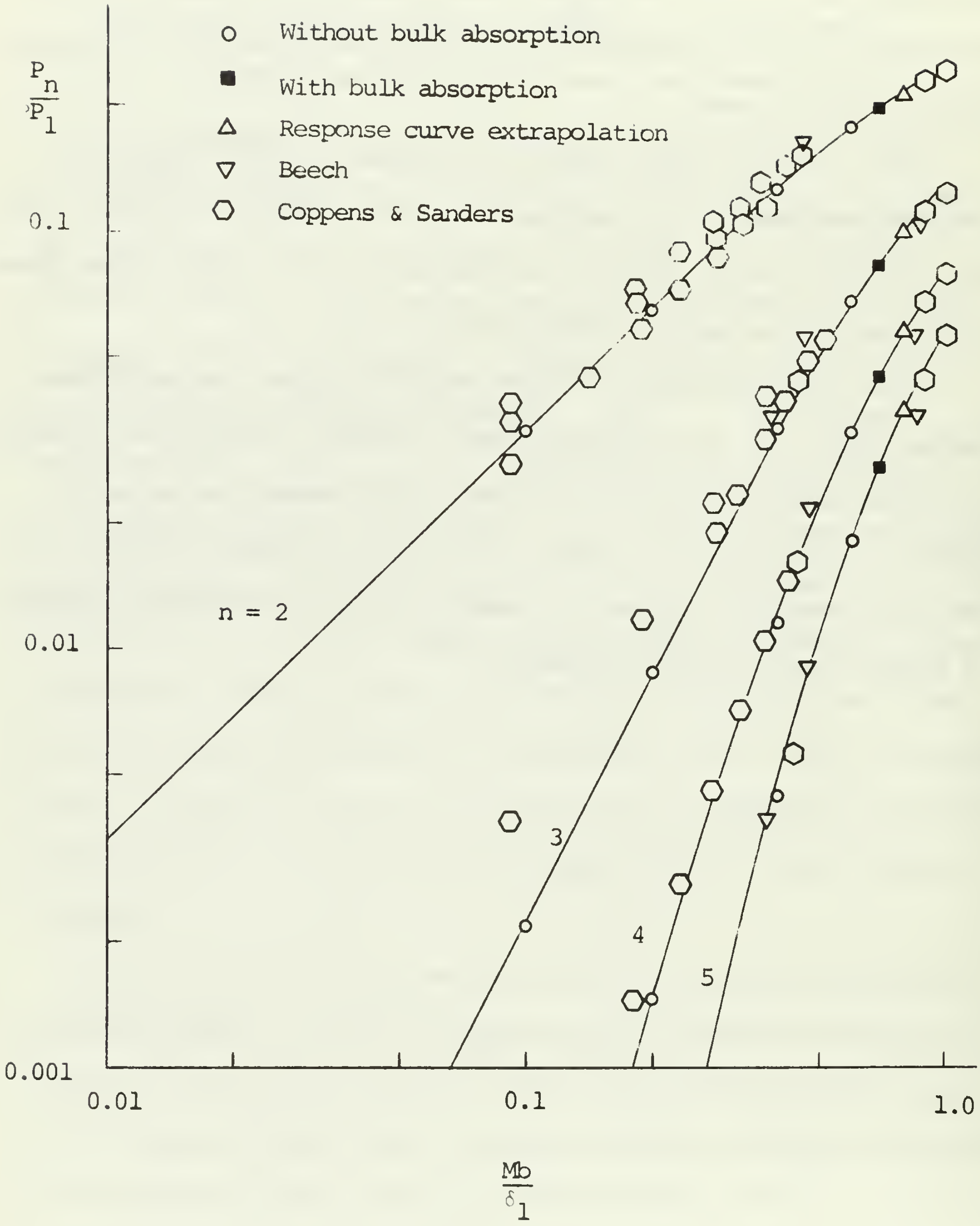
III. RESULTS, DISCUSSION, AND CONCLUSIONS

The computer program written to solve Eq. 1.1 using Eqs. 2.12, 2.13, and 2.14 gave results consistent with the theoretical predictions of Coppens and Sanders [5] and Ruff [10]. An additional perturbation solution that was performed to insure that the program was operating properly may be found in Appendix B. An initially sinusoidal pressure waveform when used with a correction factor of unity gave rapid convergence to a distorted waveform except that in the neighborhood of response parameter $\frac{2\Delta\omega}{\omega_r\delta_1}$ the solution became unstable. This region of instability gradually increased as the strength parameter was increased. For these cases the modifications defined by Eqs. 2.16 and 2.17 were used with $\varepsilon = 0.001$ and allowed a few more values of P_n/P_1 to be computed, but was not successful in generating stable results for the higher strengths. The excellent agreement with Coppens and Sanders is illustrated in Fig. 2 for the strength curves. Figures 3, 4, and 5 illustrate both the area where the solution deteriorated and also the frequency characteristics of the various harmonics.

Since the only difficulties encountered were in the vicinity of a response parameter of unity, it seems plausible that the instability is a result of the behavior of the higher harmonics which become important in this region. Recall that a response parameter of unity corresponds to the optimum excitation of the higher harmonics and that of necessity a truncated series has been used. Then the higher harmonics that are retained are affected by the absence of the harmonics that have been discarded and react by growing in amplitude to absorb the energy that would have been dissipated by those terms that have been omitted. The next refinement of the theoretical development of this research would be to cause the retained harmonics to behave as if the neglected higher harmonics were present.

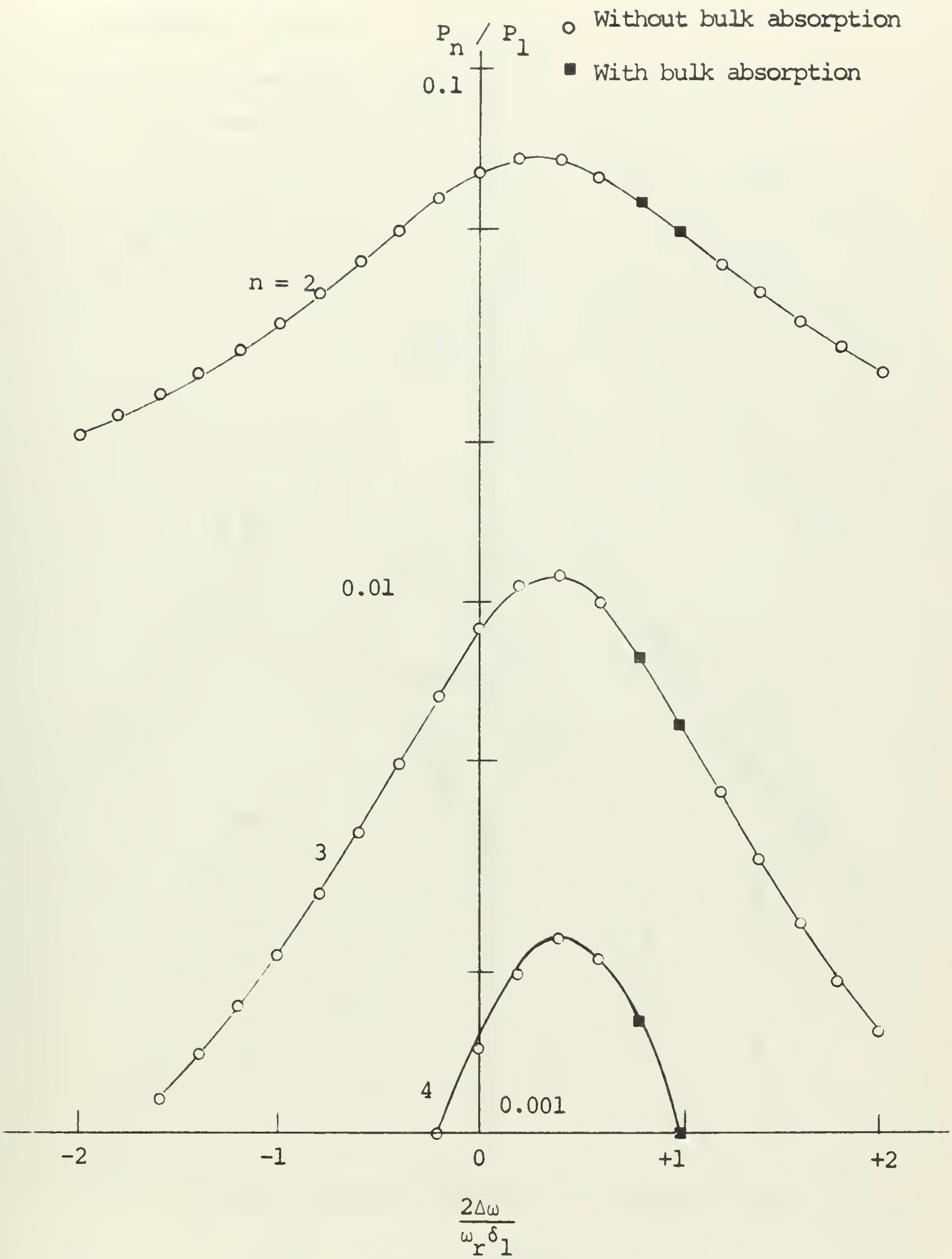
Since a solution using iterative processes is easily adapted to a computer simulation it is suggested that this method be continued. The use of a time-sharing system is recommended since it allows close control of the program and saves a great amount of time.

It has been shown that a Fourier-analysis approach to the Coppens-Sanders theory for the one-dimensional, nonlinear acoustic wave equation is readily adapted to computer simulation and extends the region of validity to the pre-shock régime. The series truncation appears to be the only obstacle to obtaining a solution valid for the entire spectrum of strength and response parameters.



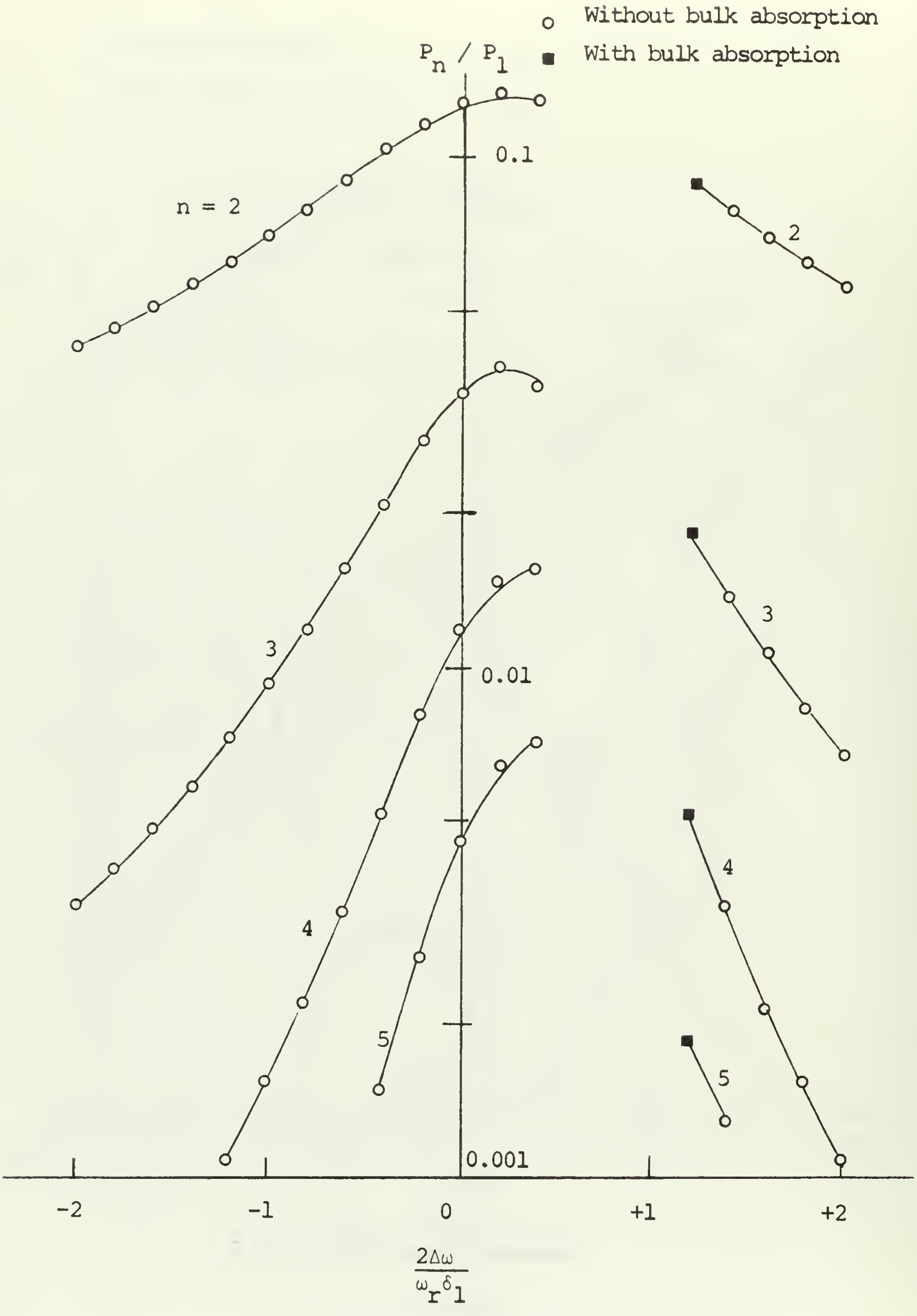
Strength Curves

Figure 2



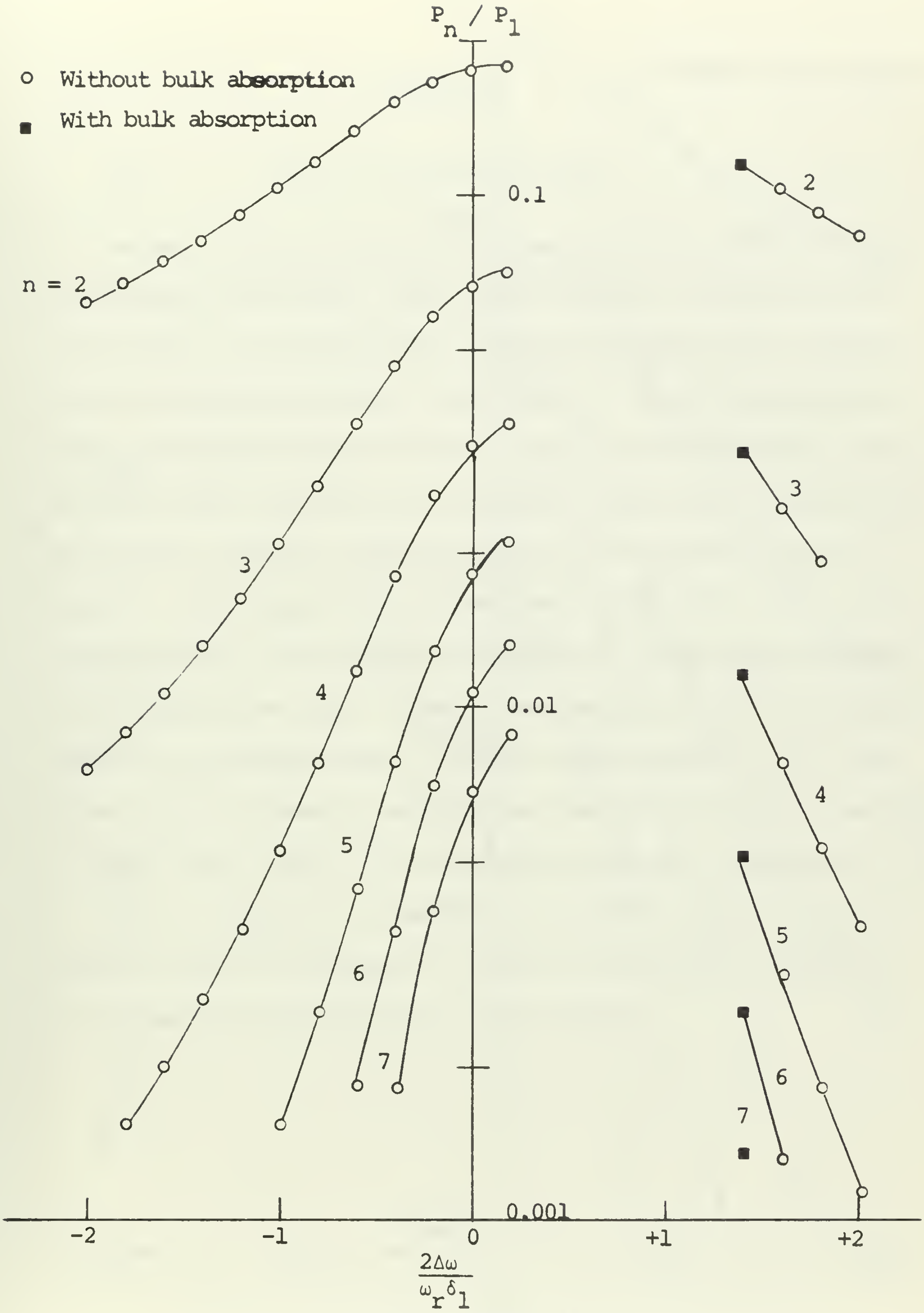
Response Curves, $Mb/\delta_1 = 0.2$

Figure 3



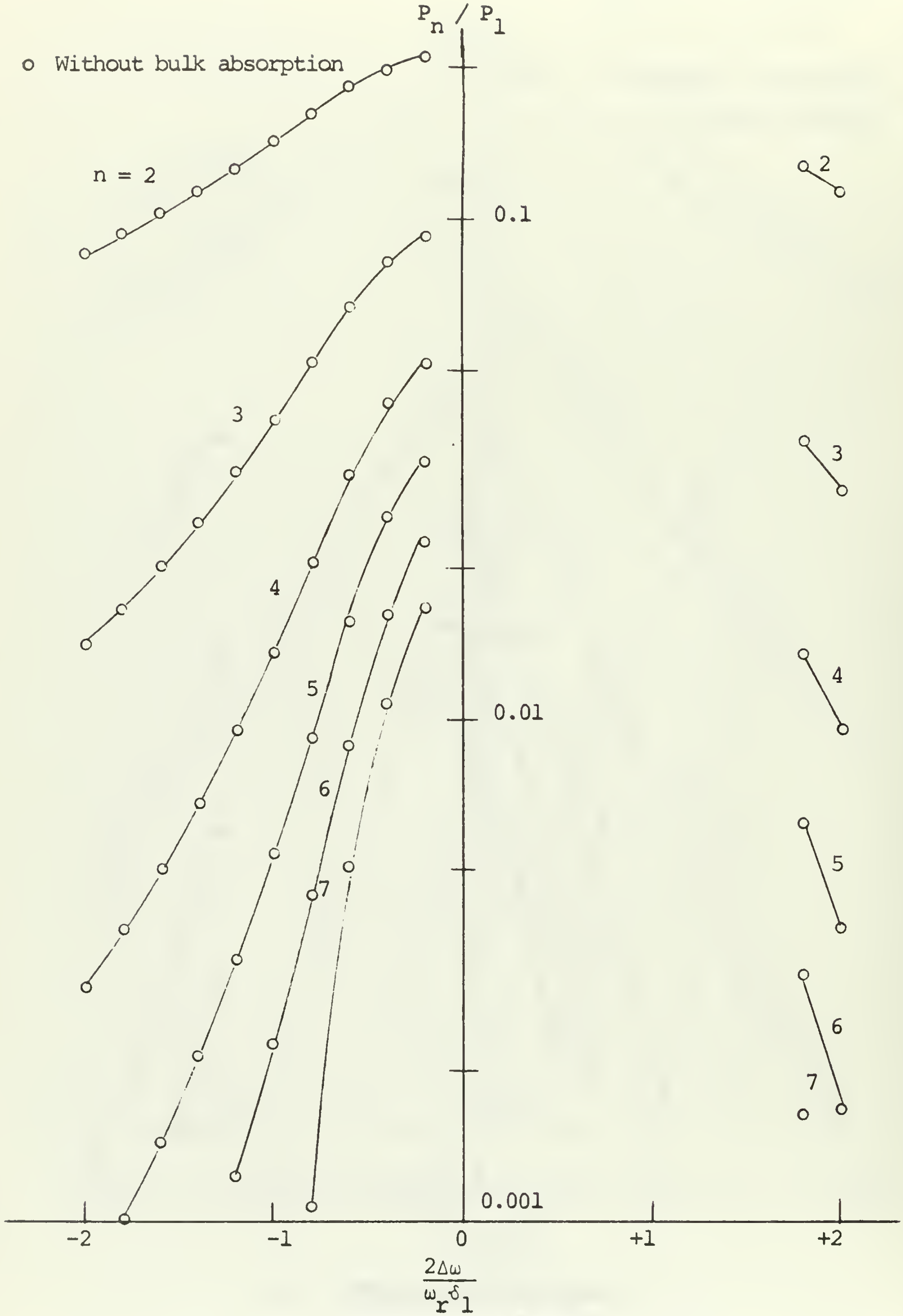
Response Curves, $Mb/\delta_1 = 0.4$

Figure 4



Response Curves, $Mb/\delta_1 = 0.6$

Figure 5



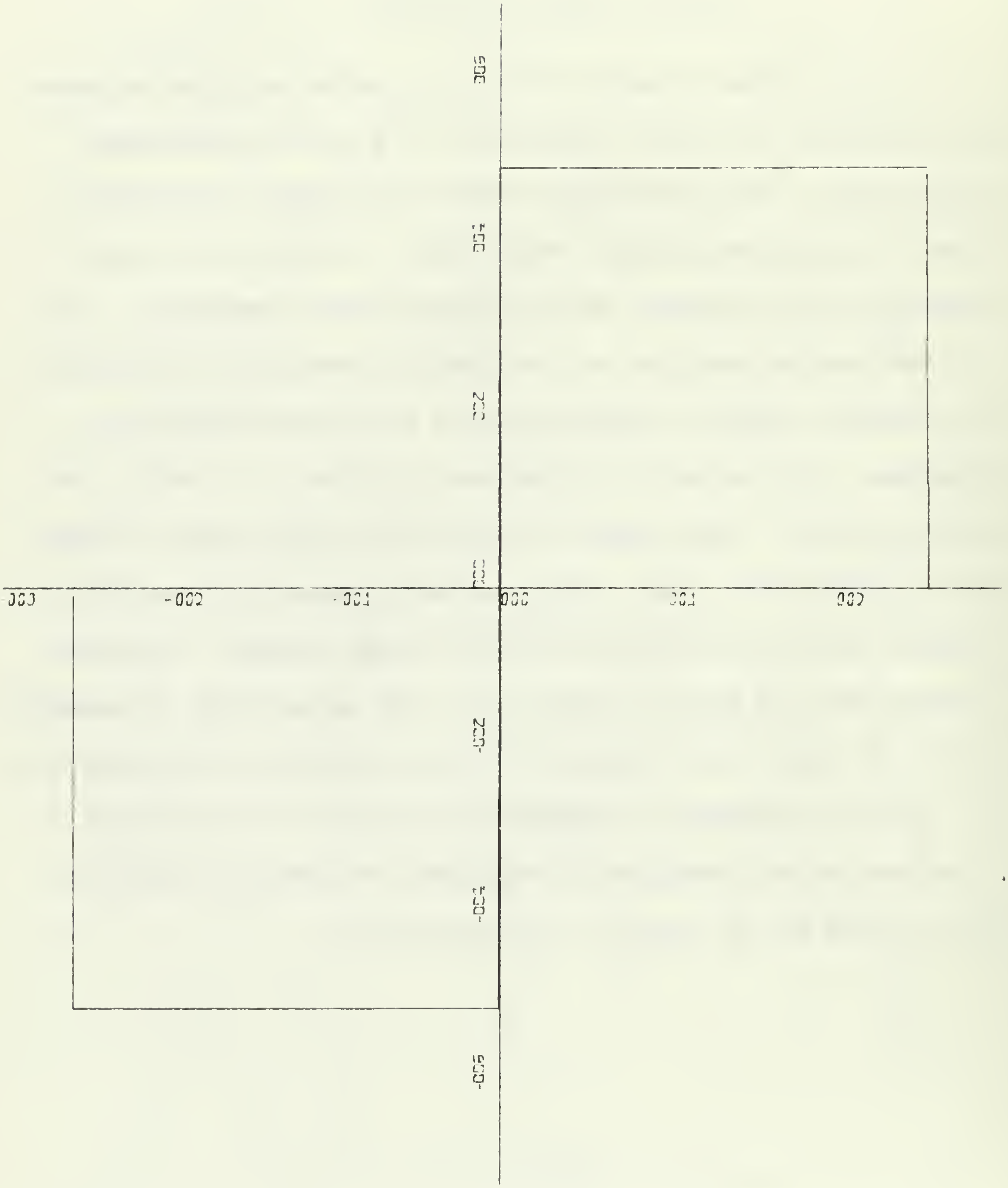
Response Curves, $Mb/\delta_1 = 0.8$

Figure 6

APPENDIX A

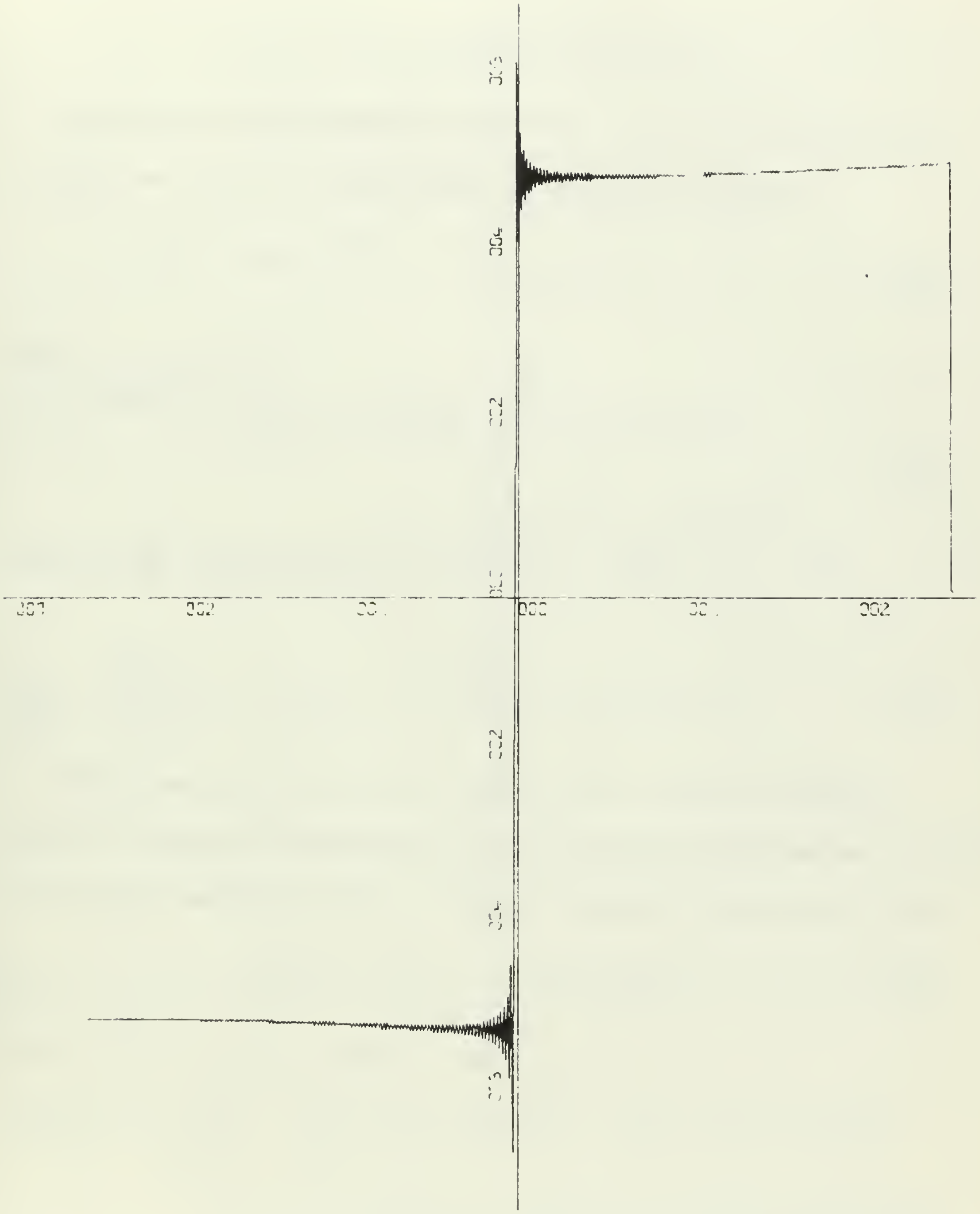
The Fast Fourier Transform

The Fast Fourier Transform [6,7,8,9] provides an effective means for estimating the Fourier coefficients of a set of evenly-spaced discrete data. The Fast Fourier coefficients differ only slightly from the conventional Fourier coefficients. Construction of waveforms is of prime interest for the higher strength parameters, where the Fast Fourier Transform has the distinct advantage of eliminating the Gaussian overshoot characteristic of the conventional Fourier Transform. This property is illustrated in Figs. 7, 8, and 9. The waveform of Fig. 7 was sampled to provide 512 evenly-spaced discrete data. This was the input to the procedures available on the IBM-360 Computer that perform Fourier and Fast Fourier analyses. The coefficients from the Fourier analysis were used to construct the waveform of Fig. 8, while those of the Fast Fourier analysis were used for Fig. 9. From the standpoint of computer utilization, the Fast Fourier Transform has the advantages of requiring less space and less time and was used for the analyses in this research.



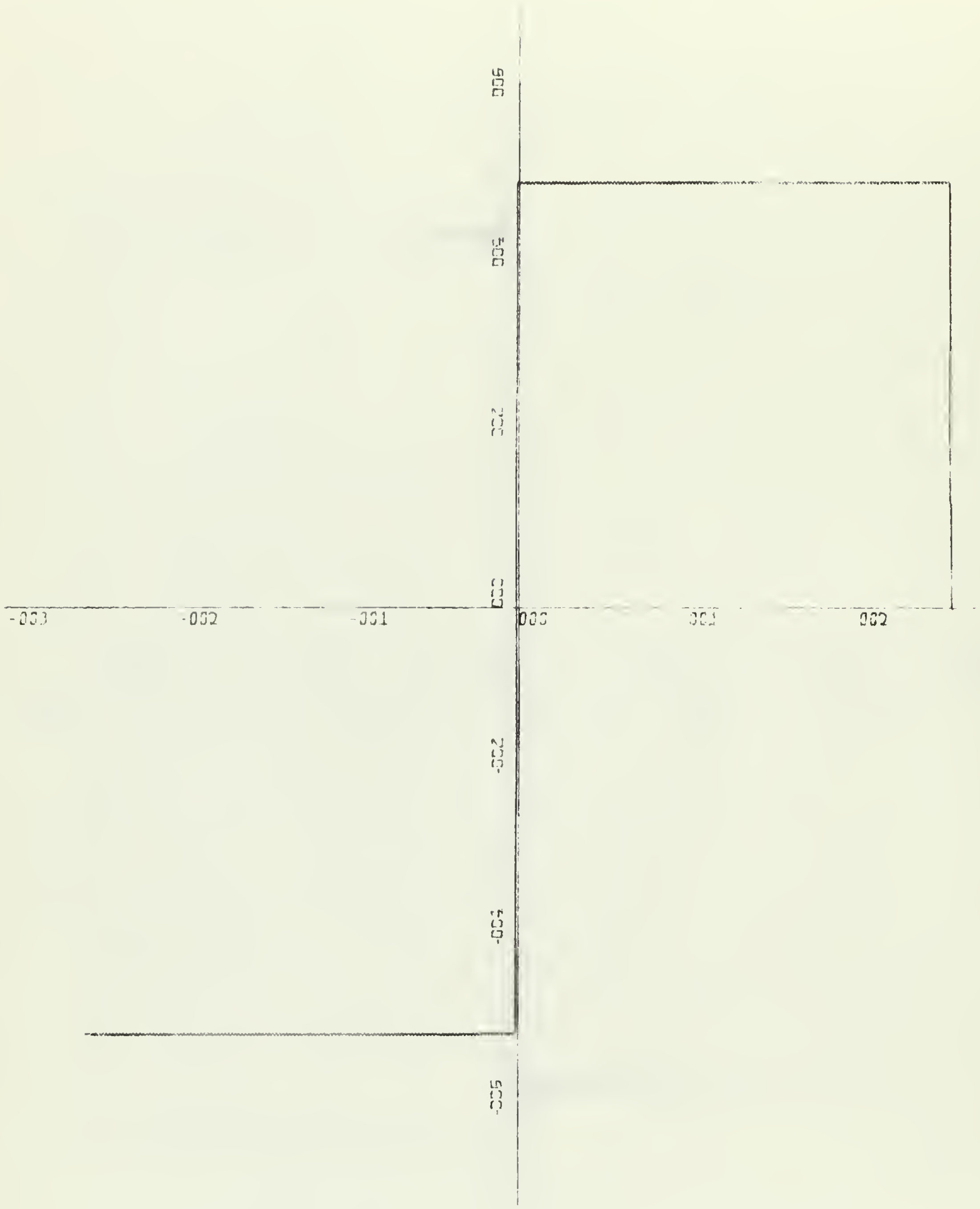
Waveform for Analysis

Figure 7



Reconstructed Waveform Using Fourier Transform

Figure 8



Reconstructed Waveform Using Fast Fourier Transform

Figure 9

APPENDIX B

Perturbation Solution of Eq. 1.1

Assume as a first approximation that the solution to Eq. 1.1 consists of only the fundamental which may be written as

$$\mathcal{P}_1 = \rho_0 c_0^2 M \cos k(L-x) \sin \omega t \,. \tag{B.1}$$

Then

$$\mathcal{P}_1^2 = \left(\frac{\rho_0 c_0^2 M}{2}\right)^2 \Big[1 + \cos 2k(L-x) - \cos 2\omega t - \cos 2k(L-x)\cos 2\omega t\Big] \,. \tag{B.2}$$

so that the right-hand side of Eq. 2.2 is, at x = L,

$$\frac{b}{\rho_0 c_0^2}\,\frac{\partial^2(\mathcal{P}_1^2)}{\partial x^2} = b \rho_0 c_0^2 M^2 k^2 \big[-1 + \cos 2\omega t\big] \,. \tag{B.3}$$

The squaring of the sin(ωt) gives a D.C. term and cos(2ωt). The D.C. terms may be excluded [5] so that the second harmonic required on the left-hand side of Eq. 2.2 may be assumed to be of the form

$$\mathcal{P}_2 = P_2 \cos 2k(L-x) \sin(2\omega t + \phi_2) \,. \tag{B.4}$$

Then $-(\Box_L^2 + \mathcal{D}_{L2})\mathcal{P}_2$ may be written as

$$-4k^2\left(-1 + \frac{\omega^2}{k^2 c_0^2} + \frac{\delta_1}{\sqrt{2}}\right) P_2 \cos 2k(L-x) \sin(2\omega t + \phi_2) + 2\sqrt{2}\, k^2 \delta_1 \cos 2k(L-x) \cos(2\omega t + \phi_2) \tag{B.5}$$

For $\Delta\omega = 0$, $\frac{\omega^2}{k^2 c_0^2} = 1 - \delta_1$, so that at x = L, Exp. B.5 may be written as

$$-4P_2 k^2 \delta_1 [(-1 + 1/\sqrt{2}) \sin(2\omega t + \phi_2) - (1/\sqrt{2}) \cos(2\omega t + \phi_2), \tag{B.6}$$

which when simplified is

$$-4P_2 k^2 \delta_1\left[.768\sin(2\omega t+\phi_2+247.5^\circ)\right] . \tag{B.7}$$

Equating Exps. B.3 and B.7

$$-4P_2 k^2 \delta_1\left[.768\sin(2\omega t+\phi_2+247.5^\circ)\right] = b\rho_o c_o^2 M^2 k^2\left[-1+\cos 2\omega t\right] \tag{B.8}$$

which may be solved for P_2 and ϕ_2. For $\frac{Mb}{\delta_1} = 0.1$,

$P_2 = .0325\rho_o c_o^2 M$ and $\phi_2 = .39$ radians.

This agrees with the predicted values of Coppens and Sanders [5] and Ruff [10] and the results of the computer program written to solve Eq. 1.1.

COMPUTER PROGRAM

This section contains the computer program used to solve Eq. 1.1 and a list of the symbology pertinent to the program.

AAMP(N) = A_{n-1}
FEE(N) = ϕ_{n-1}
BAMP(N) = B_{n-1}
GAMMA(N) = Γ_{n-1}
H(N) = H_{n-1}
THETA(N) = θ_{n-1}
P(N) = amplitudes of $p/M\rho_o c_o^2$ for increment N
R(N) = amplitudes of $(p/M\rho_o c_o^2)^2$ for increment N
ARG = $\omega t/511$
RESPM = $2\Delta\omega/\omega_r\delta_1$
STRPM = Mb/δ_1
CORR = Correction factor

Computer Program Symbology

Table I

Computer Program for Solving Eq. 1.1

```
      DIMENSION AAMP(257),FEE(257),BAMP(257),GAMMA(257),
     1H(257),THETA(257),INV(64),S(64),P(600),R(600)
      PI=3.141593
      ARG=2*PI/511.
C
C
C  INITIALIZE RESPM,STRPM,CORR,AAMP(N),FEE(N).
C
C
      READ (5,2) RESPM,STRPM,CORR
    2 FORMAT(3F6.3)
      READ(5,4) (AAMP(I),FEE(I),I=1,257)
    4 FORMAT(3(4X,2F10.7))
C
C
C  THIS SECTION CALCULATES THE AMPLITUDE OPERATORS H(N) AND
C                    THE PHASE OPERATORS THETA(N).
C
C
      DO 1 I=2,256
      J=I-1
      XN=J
      RATIO=-(1.0/SQRT(XN))
      DUMMY1=-RATIO-1.0+RESPM
C
C     EQUATION 2.13.
      H(I)=1.0/SQRT(DUMMY1*DUMMY1+RATIO*RATIO)
C
C     EQUATION 2.14.
      THETA(I)=ATAN2(RATIO,DUMMY1)
C
    1 CONTINUE
      DO 1000 NPASS=1,27
C
C
C  THIS SECTION CALCULATES THE AMPLITUDE'S BAMP(N) AND THE
C    PHASES GAMMA(N) CORRESPONDING TO THE SQUARE OF THE
C                         PRESSURE.
C
C
      DO 11 I=1,512
      P(I)=0.0
      DO 12 J=1,257
      P(I)=P(I)+AAMP(J)*SIN(ARG*(J-1)*(I-1)+FEE(J))
   12 CONTINUE
      R(I)=(P(I)*P(I))
   11 CONTINUE
C
C     AS INPUT, R(I) CONTAINS THE AMPLITUDES, FOR INCREMENT
C       "I," CORRESPONDING TO THE SQUARE OF THE PRESSURE.
C
      CALL RHARM (R,8,INV,S,IFERR)
C
C     AS OUTPUT, R(I) CONTAINS THE FAST FOURIER COEFFICIENTS
C     OF THE SQUARE OF THE PRESSURE: (A0)/2, B0=0, A1, B1,..
C                           (AN)/2, BN=0.
C     THE AN'S ARE THE COEFFICIENTS OF THE COSINE TERMS AND
C       THE BN'S ARE THE COEFFICIENTS OF THE SINE TERMS.
C
      DO 13 I=3,511,2
C     WRITE EACH COMPONENT OF THE SQUARE OF THE PRESSURE AS
C     BAMP SIN(N(INCREMENT)  +  GAMMA(N)) BY USING THE FAST
C                       FOURIER COEFFICIENTS.
      J=(I+1)/2
      BAMP(J)=SQRT(R(I+1)*R(I+1)+R(I)*R(I))
      GAMMA(J)=ATAN2(R(I),R(I+1))
   13 CONTINUE
```

```
C
C     SET D.C. TERMS TO ZERO.
C
      BAMP(1)=0.0
      BAMP(257)=0.0
      GAMMA(1)=0.0
      GAMMA(257)=0.0
C
C
C THIS SECTION ADJUSTS THE LEFT-HAND SIDE OF EQUATION 2.12
C                   TOWARDS THE RIGHT-HAND SIDE.
C
C
      DO 22 I=3,256
      AAMP(I)=AAMP(I)+CORR*(BAMP(I)*STRPM*H(I)*0.5-AAMP(I))
      FFF(I)=FFF(I)+(GAMMA(I)-FFF(I)-THETA(I))*CORR
   22 CONTINUE
C
C     SET D.C. TERMS TO ZERO.
C
      AAMP(1)=0.0
      FFF(1)=0.0
      AAMP(257)=0.0
      FFF(257)=0.0
C
C
C                       PRINT AND PUNCH OUTPUT.
C
C
      WRITE (6,34) NPASS
   34 FORMAT(' PASS NUMBER ',I5)
      DO 32 I=1,10
      J=I-1
      WRITE(6,35) J,AAMP(I),FFF(I),BAMP(I),GAMMA(I)
   35 FORMAT (I5,4F15.7)
   32 CONTINUE
      DO 31 I=11,257,10
      J=I-1
      WRITE (6,33) J,AAMP(I),FFF(I)
   33 FORMAT(I5,2F15.7)
   31 CONTINUE
 1000 CONTINUE
      WRITE (6,45) RESPM,STRPM,CORR
   45 FORMAT(10X,'RESPM=',F6.3,5X,'STRPM=',F5.3,5X,'CORR=',
     1F5.3)
      PUNCH 43 ,(I,AAMP(I),FFF(I),I=1,257)
   43 FORMAT(3(I4,2F10.7))
      STOP
      END
```

BIBLIOGRAPHY

1. Betchov, R., "Nonlinear Oscillations of a Column of Gas," Phys. Fluids, 1, 205-212 (1958).

2. Saenger, R. A. and Hudson, G. E., "Periodic Shock Waves in Resonating Gas Columns," J. Acous. Soc. Am., 32, 961-970 (1960).

3. Weston, D. E., "The Theory of the Propagation of Plane Sound Waves in Tubes," Proc. Phys. Soc. (London), 66B, 695-709 (1953).

4. Keller, J. B., "Finite Amplitude Sound Produced by a Piston in a Closed Tube," J. Acoust. Soc. Am., 26, (1954).

5. Coppens, A. B. and Sanders, J. V., "Finite-Amplitude Standing Waves in Rigid-Walled Tubes," J. Acoust. Soc. Am., 43, 516-529 (1968).

6. Titus, H., "The Fast Fourier Transform," Instructor's Notes, Naval Postgraduate School (1969).

7. Cooley, J. W. and Tukey, J. W., "An Algorithm for the Machine Calculation of Complex Fourier Series," Mathematics of Computation, 19, 297-301 (1965).

8. Gentlemen, W. M. and Sande, G., "Fast Fourier Transform for Fun and Profit," AFIPS Proc. 1966 Fall Joint Computer Conference, 29, 563-578 (1966).

9. Cochran, W. T., Cooley, J. W., Favin, D. L., Helms, H. D., Kaenel, R. A., Lang, W. W., Maling, G. C., Jr., Nelson, D. E., Radar, C. M., and Welch, P. D., "What Is the Fast Fourier Transform?," IEEE Transaction on Audio and Electroacoustics, AU-15, 44-45, (1967).

10. Ruff, P. G., III, "A Theoretical Investigation of Finite Amplitude Standing Waves in Rigid Walled Cavities," M. S. Thesis, Naval Postgraduate School (December 1967).

11. Beech, W. L., "Finite-Amplitude Standing Waves in Rigid-Walled Tubes," M. S. Thesis, Naval Postgraduate School (December 1967).

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	20
2.	Library, Code 0212 Naval Postgraduate School Monterey, California 93940	2
3.	Chief of Naval Research Office of Naval Research Washington, D. C. 20360	1
4.	Professor Alan B. Coppens Department of Physics Naval Postgraduate School Monterey, California 93940	12
5.	LT(jg) Richard M. Kadlick U. S. Naval Nuclear Power School Mare Island Naval Shipyard Vallejo, California 94592	1

Unclassified

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)*

Naval Postgraduate School
Monterey, California 93940

2a. REPORT SECURITY CLASSIFICATION

Unclassified

2b. GROUP

3 REPORT TITLE

Computer Simulation of Finite-Amplitude Standing Waves in Rigid-Walled Ducts

4. DESCRIPTIVE NOTES *(Type of report and, inclusive dates)*

Thesis, June 1969

5. AUTHOR(S) *(First name, middle initial, last name)*

Richard M. Kadlick

6. REPORT DATE June 1969	7a. TOTAL NO. OF PAGES 35	7b. NO. OF REFS 11

8a. CONTRACT OR GRANT NO.

b. PROJECT NO.

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)

9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

10. DISTRIBUTION STATEMENT

Distribution of this document is unlimited.

11. SUPPLEMENTARY NOTES

12. SPONSORING MILITARY ACTIVITY

Naval Postgraduate School
Monterey, California 93940

13. ABSTRACT

The Coppens-Sanders theory for the one-dimensional, nonlinear, acoustic wave equation with dissipative term describing the viscous and thermal energy losses encountered in a rigid-walled, closed tube of large length-to-diameter ratio was applied to finite-amplitude standing waves by the use of the Fast Fourier Transform. Computer programs were written to determine the amplitudes and phases of the first 255 harmonics. Curves of harmonic distortion as a function of the strength parameter were found to be in excellent agreement with available experimental data, to agree with the Coppens-Sanders perturbation analysis, and to extend the theoretically describable regime closer to strengths leading to the formation of the shock front.

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6811

Unclassified

Security Classification

A-31408

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Coppens-Sanders Theory Finite-amplitude Standing waves Rigid-walled ducts Fast Fourier Transform Computer Simulation						

DD FORM 1 NOV 65 1473 (BACK)
S/N 0101-807-6821

A-31409

thesK107
Computer simulation of finite amplitude
3 2768 002 11358 1
DUDLEY KNOX LIBRARY